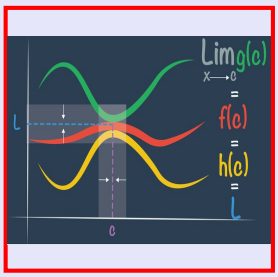
Math 261

Spring 2023

Lecture 32



Feb 19-8:47 AM

Given $f(x) = 3x^3 - 9x + 6$

1) Domain Polynomial $\rightarrow (-\infty, \infty)$

2) Is 1 a root of $f(x)$?

1	3	0	-9	6
		3	3	-6
	3	3	-6	0

Yes

3) Find all intercepts.

Y-Int $(0, 6)$

X-Int $(1, 0)$, $(-2, 0)$

Twice once

$f(x) = (x-1)(3x^2+3x-6)$

$f(x) = 3(x-1)(x^2+x-2)$

$f(x) = 3(x-1)(x-1)(x+2)$

4) $f'(x) = 9x^2 - 9$

5) $f''(x) = 18x$

6) $f'(x) = 0 \rightarrow x = \pm 1$

7) $f''(x) = 0 \rightarrow x = 0$

Apr 12-8:48 AM

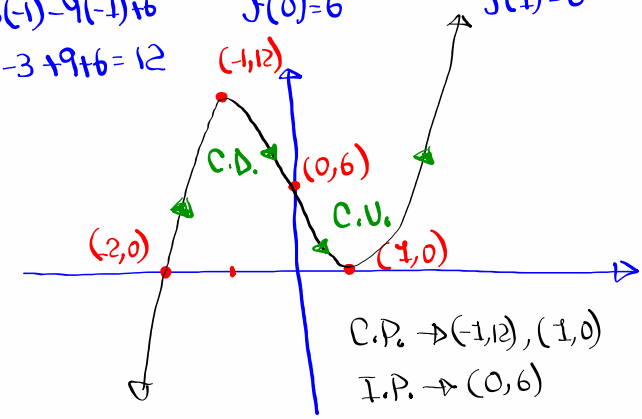
8)

x	$-\infty$		-1		0		1		∞
$f'(x)$		$+$	0	$-$		$-$	0	$+$	
$f''(x)$		$-$		$-$	0	$+$		$+$	
$f(x)$		↗		↘		↘		↗	

$f(-1)=3(-1)^3-9(-1)+6$
$=-3+9+6=12$

$f(0)=6$

$f(1)=0$



C.P. → (-1,12), (1,0)
I.P. → (0,6)

Apr 12-8:54 AM

$f(x)=\dfrac{x^2-1}{x^3}$

1) Domain $x\neq 0$ $(-\infty,0)\cup(0,\infty)$

2) $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{\frac{x^2}{x^3}-\frac{1}{x^3}}{\frac{x^3}{x^3}}=\boxed{0}$

3) $\lim\limits_{x\to-\infty} f(x)=\boxed{0}$

3) All asymptotes V.A. → $x=0$
H.A. → $y=0$

4) All intercepts No Y-Int. since $x\neq 0$
x-Int. → $y=0$ → $f(x)=0$ → $x^2-1=0$
$x=\pm 1$
→ $(\pm 1,0)$

5) Is $f(x)$ even, or odd?

$f(-x)=\dfrac{(-x)^2-1}{(-x)^3}=\dfrac{x^2-1}{-x^3}=-\dfrac{x^2-1}{x^3}=-f(x)$

odd function
symmetric w/r origin.

Apr 12-9:01 AM

$$f(x) = \frac{x^2-1}{x^3} = \frac{x^2}{x^3} - \frac{1}{x^3} = \frac{1}{x} - \frac{1}{x^3} = x^{-1} - x^{-3}$$

6) $f'(x) = -1x^{-2} + 3x^{-4} = x^{-4}(3-x^2) = \dfrac{3-x^2}{x^4}$

7) $f''(x) = 2x^{-3} - 12x^{-5} = 2x^{-5}(x^2-6) = \dfrac{2(x^2-6)}{x^5}$

$f'(x) = 0 \rightarrow x = \pm\sqrt{3}$

$f'(x)$ is undefined at $x = 0$

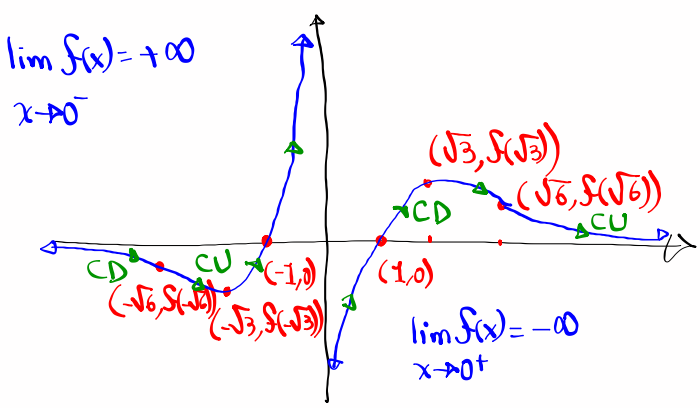
$f''(x) = 0 \rightarrow x = \pm\sqrt{6}$

$f''(x)$ is undefined at $x = 0$

Apr 12-9:09 AM

$f'(x) = \dfrac{3-x^2}{x^4}$, $f''(x) = \dfrac{2(x^2-6)}{x^5}$

x	$-\infty$	$-\sqrt{6}$	$-\sqrt{3}$	0		$\sqrt{3}$		$\sqrt{6}$	∞
$f'(x)$			0	①	+	0	−		−
$f''(x)$		0		①	−		−	0	+
$f(x)$					↗		↘		↘

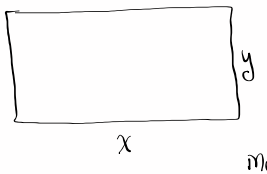


Apr 12-9:15 AM

I have 100 meters of fencing.

I want to have an enclosed rectangular area for my dog.

Find dimensions that gives me maximum area.



$2x + 2y = 100$

$x + y = 50$

Area xy

Area $x(50-x)$

$f(x) = 50x - x^2$

$f'(x) = 50 - 2x$

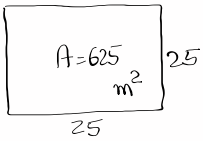
$f''(x) = -2 < 0$ C.D.

$f'(x) = 0$

$50 - 2x = 0$

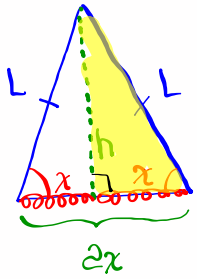
$\boxed{x = 25}$

$\boxed{y = 25}$



Apr 12-9:25 AM

An isosceles triangle has two equal sides with length L.



Show that to have max. area, we must have a right triangle.

$$\text{Area} = \frac{\text{base} \cdot \text{height}}{2}$$

$$= \frac{2x \cdot h}{2} = xh$$

Pythagorean thrm

$x^2 + h^2 = L^2$

$h^2 = L^2 - x^2$ $\quad h = \sqrt{L^2 - x^2}$

$\text{Area} = x\sqrt{L^2 - x^2}$

$f(x) = x(L^2 - x^2)^{1/2}$

Find $f'(x)$, Solve $f'(x) = 0$, use example from yesterday to determine max or min. pt.

Apr 12-9:31 AM

Class QZ 9

Use Calc. method to draw $f(x) = -x^3 + 3x^2 + 2$

No need for exact intercepts.

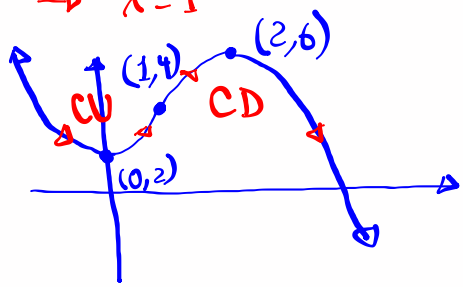
$f'(x)$, $f''(x)$, chart

$f(0) = 2$
$f(1) = 4$
$f(2) = 6$

$f'(x) = -3x^2 + 6x$ $\quad f'(x) = 0 \rightarrow x = 0, 2$

$f''(x) = -6x + 6$ $\quad f''(x) = 0 \rightarrow x = 1$

x	$-\infty$	0	1	2	∞
$f'(x)$	$-$	0	$+$	0	$-$
$f''(x)$	$+$	$+$	0	$-$	$-$
$f(x)$	↘	↗	↗	↘	



Apr 12-9:41 AM